

# Project : Introduction

## Objectif

The objective of this project is to design and implement a parallel approach for solving the heat equation in 2D.

## 1 – Domain decomposition

We consider the linear operator  $\mathcal{L}$ . Let us divide the plane in  $x$  direction into two subdomains  $\Omega_1 = [0, \Gamma_2]$  and  $\Omega_2 = [\Gamma_1, 1]$ . To simplify the following demonstration, we will consider that  $\Gamma_1$  is at  $x = \frac{L}{2}$ , and  $\Gamma_2$  is at  $x = -\frac{L}{2}$ , where  $L$  is the size of the overlap between  $\Omega_1$  and  $\Omega_2$ .

We consider the following equations with bounded boundary conditions at infinity :

$$\begin{cases} \mathcal{L}(u) = -\Delta(u) = 0 \\ |u| < \infty \quad |x| \rightarrow \infty \end{cases} \quad (1)$$

The problems to solve on each subdomain are the followings :

$$\begin{cases} L[u_1^{n+1}] = f_1 \text{ in } \Omega_1 \\ u_1^{n+1}|_{\partial\Omega \cap \partial\Omega_1} = g_1 \text{ on } \partial\Omega \cap \partial\Omega_1 \\ u_1^{n+1}|_{\partial\Gamma_1} = u_2^n|_{\partial\Gamma_1} \end{cases} \quad (2)$$

and

$$\begin{cases} L[u_2^{n+1}] = f_2 \text{ in } \Omega_2 \\ u_2^{n+1}|_{\partial\Omega \cap \partial\Omega_2} = g_2 \text{ on } \partial\Omega \cap \partial\Omega_2 \\ u_2^{n+1}|_{\partial\Gamma_2} = u_1^n|_{\partial\Gamma_2} \end{cases} \quad (3)$$

where  $\Gamma_1$  and  $\Gamma_2$  denote respectively the virtual interface  $\Omega_1$  and  $\Omega_2$ , and  $\Omega_2$  and  $\Omega_1$ .

We supposed that the problem is well posed on each subdomain

## 2 – Framework for the Aitken-Schwarz method

Let us consider a linear differential problem

$$\begin{cases} L[U] = f \text{ in } \Omega \\ U|_{\partial\Omega} = g \end{cases} \quad (4)$$

We construct an iterative method by dividing the original domain into  $N$  subdomains. We restrict ourselves to two overlapping subdomains  $\Omega_1$  and  $\Omega_2$ , where  $\Omega_1 \cup \Omega_2 = \Omega$

For the additive Schwarz algorithm, we construct an iterative procedure as follows. Starting with an initial guess for the solution on the interfaces, problems on the subdomains only are solved. The boundary data at the interface are obtained from a previous iterate using Dirichlet transmission condition. Each subdomain can be solved in sequential.

## 3 – Description of the task

The task is to apply the Schwarz domain decomposition method with overlap to solve the Laplace equation in 2D with the given boundary conditions in the file. The code should make use of OpenMP.

The starting sequential code is given on the course page.